Simulations of Fluid Dynamics and Heat Transfer in $LH_2$ Absorbers

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Introduction: Approaches to Heat Removal

Two approaches under consideration:

1. **External cooling loop** (traditional approach).
   - Bring the $LH_2$ to the coolant (heat removed in an external heat exchanger).

2. **Combined absorber and heat exchanger.**
   - Bring the coolant, i.e. $He$, to the $LH_2$ (remove heat directly within absorber).
Advantages/disadvantages of an external cooling loop:

+ Has been used for several $LH_2$ targets (e.g. SLAC E158).
+ Easy to regulate bulk temperature of $LH_2$.
+ Is likely to work best for small aspect ratio ($L/R$) absorbers.

− May be difficult to maintain uniform vertical flow through the absorber.

Advantages/disadvantages of a combined absorber/heat exchanger:

+ Takes advantage of natural convection transverse to the beam path.
+ Flow in absorber is self regulating, i.e. larger heat input ⇒ more turbulence ⇒ enhanced thermal mixing.
+ Is likely to work best for large aspect ratio ($L/R$) absorbers.

− More difficult to ensure against boiling at very high Rayleigh numbers.
Heat Exchanger Analysis

Energy balance between $LH_2$ and coolant ($He$).

✔ Parameters:

\[
\begin{align*}
T_i & = \text{coolant inlet temperature} \\
T_o & = \text{coolant outlet temperature} \\
T_{LH_2} & = \text{bulk temperature of } LH_2 \\
A & = \text{surface area of cooling tubes} \\
h_{LH_2} & = \text{convective heat transfer coefficient of } LH_2 \\
h_{He} & = \text{convective heat transfer coefficient of } He \\
\Delta x & = \text{thickness of cooling tube walls} \\
k_w & = \text{thermal conductivity of cooling tube walls} \\
c_p & = \text{specific heat capacity of } He
\end{align*}
\]
Rate of heat transfer:

\[
\dot{q} = -\frac{A(T_o - T_i)}{\left( \frac{1}{h_{LH_2}} + \frac{\Delta x}{k_w} + \frac{1}{h_{He}} \right) \ln \left( \frac{T_{LH_2} - T_o}{T_{LH_2} - T_i} \right)}
\]

Mass flow rate of He:

\[
\dot{m}_{He} = \frac{\dot{q}}{c_p (T_o - T_i)}.
\]

\(h_{He}\) \(\Rightarrow\) from appropriate correlation (flow through a tube).
\(h_{LH_2}\) and \(T_{LH_2}\) \(\Rightarrow\) from CFD simulations (no correlations for natural convection with heat generation).
Features of the CFD Simulations:

✓ Provides average convective heat transfer coefficient and average $LH_2$ temperature for heat exchanger analysis.

✓ Track maximum $LH_2$ temperature (cf. boiling point).

✓ Determine details of fluid flow and heat transfer in absorber.

⇒ Better understanding leads to better design!
Take 1: Results using **FLUENT** (M. Boghosian):

- Simulate one half of symmetric domain.
- Steady flow calculations.
- Heat generation via *steady* Gaussian distribution.
- Turbulence modeling (RANS) used for $Ra \geq 4 \times 10^9$.

Take 2: Results using **COA code** (A. Obabko and E. Almasri):

- Simulate full domain.
- Unsteady flow calculations.
- All scales computed for all Rayleigh numbers.
  - Investigate startup behavior, *e.g.* startup overshoot in $T_{max}$.
  - Investigate possibility of asymmetric flow oscillations.
  - Investigate influence of beam pulsing.
Formulation

Properties and parameters:

- $R$ = radius of absorber
- $T_w$ = wall temperature of absorber
- $\dot{q}'''(r)$ = rate of volumetric heat generation (Gaussian distribution)
- $\dot{q}'$ = rate of heat generation per unit length
- $\nu$ = kinematic viscosity of $LH_2$
- $\alpha$ = thermal diffusivity of $LH_2$
- $k$ = thermal conductivity of $LH_2$
- $\beta$ = coefficient of thermal expansion of $LH_2$
Governing Equations ($T - \omega - \psi$ formulation)

Energy equation:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + q(r)$$

Vorticity-transport equation:

$$\frac{\partial \omega}{\partial t} + v_r \frac{\partial \omega}{\partial r} + \frac{v_\theta}{r} \frac{\partial \omega}{\partial \theta} = Pr \left[ \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \omega}{\partial \theta^2} \right] + Ra_R Pr \left[ \sin \theta \frac{\partial T}{\partial r} + \frac{\cos \theta}{r} \frac{\partial T}{\partial \theta} \right]$$

Streamfunction equation:

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = -\omega$$

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v_\theta = -\frac{\partial \psi}{\partial r}$$
Initial and boundary conditions:

\[ T = \omega = \psi = v_r = v_\theta = 0 \quad \text{at} \quad t = 0, \]
\[ T = \psi = v_r = v_\theta = 0 \quad \text{at} \quad r = 1. \]

Non-dimensional variables:

\[ r = \frac{r^*}{R}, \quad v_r = \frac{v_r^*}{R/\alpha}, \quad v_\theta = \frac{v_\theta^*}{R/\alpha}, \quad t = \frac{t^*}{R^2/\alpha}, \]
\[ T = \frac{T^* - T_w}{\dot{q}'/k}, \quad \psi = \frac{\psi^*}{\alpha}, \quad \omega = \frac{\omega^*}{\alpha/R^2}, \]
\[ q(r) = \frac{\dot{q}'''(r)}{\dot{q}'/R^2} = \frac{1}{2\pi\sigma^2}e^{-\frac{r^2}{2\sigma^2}}, \quad \sigma = \frac{\sigma^*}{R}. \]
Formulation – Non-Dimensional Parameters

Prandtl Number:

\[ Pr = \frac{\nu}{\alpha} \]

Rayleigh Number:

\[ Ra_R = Gr Pr = \frac{gR^3 \beta q' / k}{\nu \alpha} \left( = \frac{\pi}{32} Ra_{MB} \right) \]

Nusselt number:

\[ Nu_R = \frac{h_{LH_2} R}{k} \left( = \frac{Nu_{MB}}{2} \right) \]
Based on preliminary results, the following flow regimes are observed:

- **Steady, symmetric solutions**: \( Ra_R \leq 1 \times 10^8 \)
- **Unsteady, asymmetric solutions**: \( Ra_R \geq 1 \times 10^9 \)

**Steady, symmetric** results for \( Ra_R = 1.57 \times 10^7 \) (uniform heat generation):

- Streamfunction:
- Temperature:
- Vorticity:
Steady, Symmetric Results (cont’d)

Nusselt number versus $\theta$ for $Ra_R = 1.57 \times 10^7$ (uniform heat generation):

$Nu$ vs. $\theta$: 

\[Nu\ vs.\ \theta:\]
Uniform heat generation ($\sigma \rightarrow \infty$) with $Pr = 1$:

<table>
<thead>
<tr>
<th>$Ra_R$</th>
<th>Mitachi et al.</th>
<th>FLUENT</th>
<th>COA Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.57 \times 10^6$</td>
<td>8.58</td>
<td>7.7</td>
<td>8.2</td>
</tr>
<tr>
<td>$1.57 \times 10^7$</td>
<td>14.0</td>
<td>11.9</td>
<td>12.0</td>
</tr>
</tbody>
</table>

1 Mitachi et al. (1986, 1987) - Results shown are from numerical simulations which compared favorably with experiments.

2 From M. Boghosian’s correlation for $Pr = 1.4$, i.e. $\bar{N}u_{MB} = 0.7041 \cdot Ra_{MB}^{0.1864}$. 
Steady, Symmetric Results: $Ra_R = 1 \times 10^8, \sigma = 0.25$
Unsteady, Asymmetric Results: $Ra_R = 1 \times 10^9, \sigma = 0.25$

$t = 0.2$

Streamfunction:

Vorticity:

Movies for streamfunction, temperature and vorticity ($0 \leq t \leq 0.25$).
Unsteady, Asymmetric Results: $Ra_R = 1 \times 10^9, \sigma = 0.25$

Asymmetric oscillation does not significantly influence wall heat transfer (e.g. $Nu$ for left and right walls superimposed).

$t = 2.0$: 

![Graph showing unsteady, asymmetric results for $Ra_R = 1 \times 10^9, \sigma = 0.25$, with a peak at $t = 2.0$.](attachment:image.png)
Unsteady Results – High-$Ra$ Startup

Movie for $Ra_R = 1 \times 10^{11}$ ($\sigma = 0.25$).

$Nu$ vs. $\theta$:

$t = 0.0015$
Current and Future Efforts

Current and future work:

➢ Simulate Argonne test case and compare results.

➢ Determine critical Rayleigh number above which solutions are unsteady and asymmetric.

➢ Evaluate influence of $\sigma$, i.e. ratio of beam size to absorber size, on heat transfer.

➢ Obtain solutions at higher Rayleigh numbers (target $Ra_R \sim 10^{14}$).

➢ Compare high-Rayleigh number COA solutions (unsteady) with FLUENT results (steady RANS).

➢ Examine need for heater, e.g. to combat start-up overshoot.

➢ Investigate influence of pulsed beam on fluid dynamics and heat transfer.

Note that at 15 Hz, one pulse corresponds to $2.4 \times 10^{-7}$ non-dimensional time units (cf. $\Delta t = 10^{-8}$).