THE BEAM-BEAM INTERACTION IN THE PRESENCE OF STRONG RADIATION DAMPING

R. Assmann and K. Cornelis, CERN, Geneva, Switzerland

Abstract

The beam-beam interaction in electron-positron storage rings depends strongly on the radiation damping. It has been shown before that the achievable beam-beam tune shift (the beam-beam limit) is a function of the damping decrement (the damping rate per beam-beam interaction). The LEP collider has been operated and has delivered luminosity in the range of 45 GeV to 101 GeV. The beam-beam performance data from LEP is revisited and fitted with a simple model. The scaling of the beam-beam limit with the damping decrement is estimated.

1 INTRODUCTION

The LEP collider has explored the beam-beam interaction over a wide range of beam energies, from 45 GeV to recently 104 GeV. Vertical beam-beam parameters ξ_y of 0.083 per interaction point (IP) were achieved at high energy without reaching the beam-beam limit. The beam-beam effect in LEP has been discussed in numerous papers [1, 2, 3, 4, 5] and it is beyond the scope of this paper to review the existing literature. Here we compare the observed functional dependence of ξ_y on beam intensity with the expected behaviour from a simple stochastic model. The theory assumes strong radiation damping and no correlation between the effects of successive beam-beam interactions. The calculated functional relationship between ξ_y and bunch current is compared to experimental observations at high energy. The functional relationship is then used to fit for the unperturbed emittance and the beam-beam limit. Finally, the scaling of the beam-beam limit with energy is estimated.

2 THEORY

The vertical beam-beam parameter ξ_y is calculated from the measured luminosity L, the beta function β_y^* at the IP, the beam energy E, and the bunch current i:

$$\xi_y = \frac{2\, r_e e\, m_e c^2 \cdot \beta_y^*}{n_b \cdot i \cdot E} \cdot L \tag{1}$$

The term n_b denotes the number of bunches, r_e, e and m_e are the classical radius, charge and mass of the electron, and c is the light velocity. The ξ_y is related to the vertical beam-beam tune shift ΔQ_y [6,7]. With a typical value for the vertical tune in LEP ($Q_y \sim 0.18$) the tune shift is up to 10% smaller than the measured ξ_y.

The maximum value for ξ_y is in practise limited by intensity dependent beam-beam blow-up. It has been observed before that the beam-beam limit ξ_y^∞ is a function of the transverse damping time τ, the revolution frequency f_{rev} and the number n_{IP} of interaction points [8]:

$$\xi_y^\infty = \mathrm{f}\left[\lambda_d\right] = \mathrm{f}\left[\frac{1}{f_{rev} \cdot \tau \cdot n_{IP}}\right] \tag{2}$$

The damping decrement is denoted by λ_d. The functional dependence is unknown. From experimental data the following parameterisation was suggested [9]:

$$\xi_y^\infty \propto \lambda_d^{0.3} \tag{3}$$

With the feasible beam intensities LEP does not reach the beam-beam limit at high energies. However, the experimental LEP data can be used to infer the asymptotic beam-beam limit.

We used a simple stochastic theory of the beam-beam interaction to derive the following relationship between the vertical beam-beam parameter and bunch current i:

$$\xi_y = \sqrt{\frac{1}{A + (B \cdot i)^2}} \cdot i \tag{4}$$

This equation does only contain two free parameters *A* and *B*. Their physical interpretation is discussed later. The theory assumes no correlation between subsequent beam-beam interactions and is only expected to be true with strong radiation damping. Resonant beam-beam effects and the difference between the beam-beam tune shift and ξ_y are neglected. We note, that beam tails and lifetime reductions due to resonant beam-beam effects are not visible during high energy operation of LEP.

The physical interpretation of the free parameters *A* and *B* is now discussed. If there is **no beam-beam blow-up** then *B*=0 and *A* is:

$$A = \left(\frac{2\pi\ e\, f\, \gamma}{r_e}\right)^2 \cdot \frac{\beta_x^*}{\beta_y^*} \cdot \varepsilon_x^0 \cdot \varepsilon_y^0 \tag{5}$$

A is a machine and optics dependent number times the zero current (unperturbed) vertical emittance ε_y^0 (if the horizontal beam-beam blow-up is small). Indeed we can use it to calculate the unperturbed vertical emittance. The slope of ξ_y with i is mainly determined by ε_y^0:

$$\xi_y = \sqrt{\frac{1}{A}} \cdot i \tag{6}$$

Note that ξ_y is zero for zero bunch current. This is an important constraint when analysing the LEP data. ξ_y becomes independent of beam current in the **beam-beam**

limit. Then B gives the inverse asymptotic vertical beam-beam parameter:

$$B = \frac{1}{\xi_y(i \to \infty)} \tag{7}$$

If the term B is not equal zero then ξ_y will start to saturate at some bunch current.

3 LEP MEASUREMENTS AND FITS

During the LEP runs almost all parameters and observables of the machine performance are logged for the later analysis. In particular we can analyse the dependence of ξ_y as a function of beam intensity. Equation (4) can be fitted to the data and allows extracting the asymptotic ξ_y.

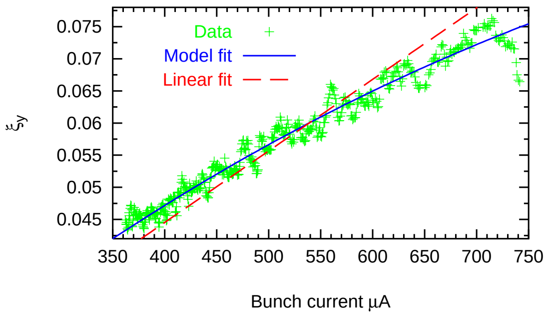

Figure 1: Measured ξ_y at 94.5 GeV versus bunch current. The data is fitted with ("Model fit") and without ("Linear fit") beam-beam limitation.

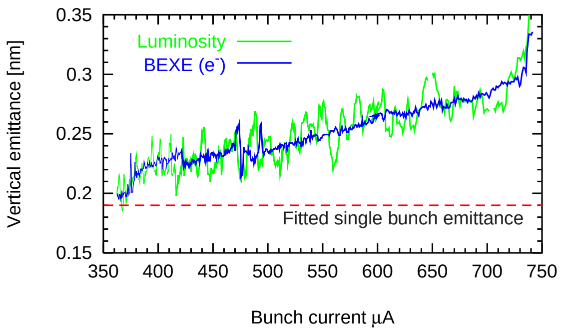

Figure 2: The vertical emittance as fitted and calculated from luminosity and synchrotron beam size measurements (BEXE).

Figure 1 shows the observed ξ_y at 94.5 GeV and compares it both with a fit using Equation (4) ("Model fit") and a linear fit (B=0) for fill 5259. The beam-beam fit explains the ξ_y dependence on current with an unperturbed vertical emittance of 0.19 nm [A = (6.12 ± 0.05) 10^7 µA^2] and an asymptotic ξ_y of 0.122 [B = 8.18 ± 0.11].

In order to verify that the observed saturation in ξ_y is correctly attributed to the beam-beam effect, we consider the current dependent blow-up of the vertical emittance. The luminosity data allows calculating the vertical emittance, assuming that the optical functions are known and that the horizontal emittance has its design value. The data can be compared with the measured vertical beam size in the BEXE instrumentation [10]. Figure 2 shows that the vertical emittance blow-up calculated from the measured luminosity is consistent with the blow-up observed from the BEXE measurement of beam size. At lower bunch current the vertical emittance approaches its fitted unperturbed value.

The asymptotic vertical beam-beam parameter was constrained to 0.122 and the term A was fitted to average data of July and August 1998. The result is shown in the Figure 3. The difference in performance can be explained by an unperturbed vertical emittance of 0.19 nm in October and 0.27 nm in July and August 1998.

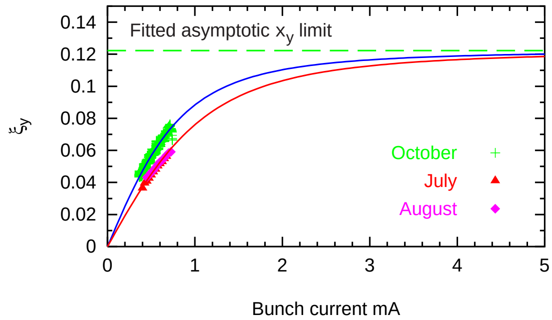

Figure 3: Three data sets at 94.5 GeV are fitted with the constraint of equal asymptotic beam-beam parameter ξ_y

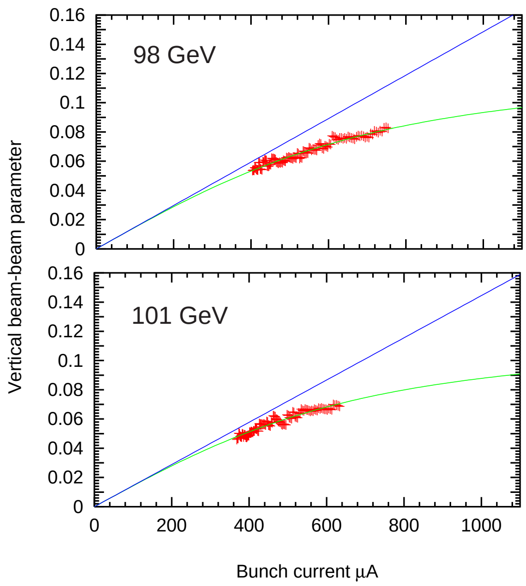

Figure 4: Examples of fitted ξ_y versus current for 98 and 101 GeV. The straight line shows the non beam-beam limited behaviour assuming the fitted unperturbed vertical emittance.

Beam-beam fits to data from 98 GeV and 101 GeV are shown in Figure 4. The fits suggest a beam-beam limit of 0.115/0.111 and an unperturbed vertical emittance of 0.108/0.082 nm (98/101 GeV). By applying the described method, we consistently find a beam-beam limit around 0.11-0.12 at high energy. Note that an alternative origin of the beam-beam limit was studied in [11]. If the beam-

beam interaction shifts the horizontal tune towards the half integer resonance, the horizontal beam size is blown-up and the beam-beam parameter effectively limited. The beam-beam limit was estimated to be around 0.111 for this case. The measurements of vertical emittance blow-up (Figure 2) indicate that this limit has not yet been reached for LEP. Some beam-beam blow-up in the horizontal beam size has been observed, but measurements show that it is smaller than the blow-up in the vertical plane.

The model of the ξ_y variation with beam current allows estimating the ultimate performance limit of LEP for a given beam energy and intensity. In Figure 5 it is shown that an ultimate peak luminosity of 1.5 10^{32} cm^{-2} s^{-1} is predicted at 98 GeV and 6 mA total beam intensity in 8 bunches (with vanishing vertical emittance at zero current). This must be compared to the achieved peak luminosity of 10^{32} cm^{-2} s^{-1}.

.

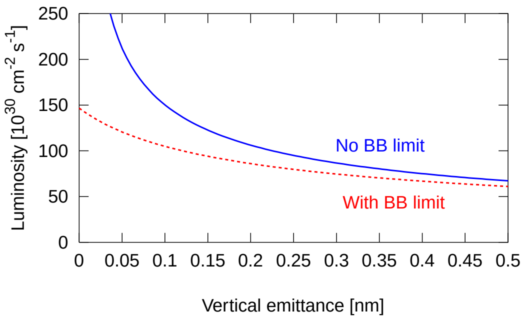

Figure 5: Predicted luminosity versus unperturbed vertical emittance (emittance at zero beam intensity). The calculation assumes a beam energy of 98 GeV and a bunch current of 750 µA. It is based on the fitted beam-beam limit of 0.115 for 98 GeV (Figure 4, top).

Table 1: Overview of achieved beam energies, ξ_y, bunch currents, and transverse damping times in LEP.

Year	Beam energy [GeV]	Maximum ξ_y	Damping time [turns]	Bunch current [µA]
1994	45.6	0.045	721	320
1995	65.0	0.050	249	400
1996	86.0	0.040	107	525
1997	91.5	0.055	89	650
1998	94.5	0.075	81	750
1999	98.0	0.083	73	780
2000*	102.7	0.055	63	550

4 SCALING OF BEAM-BEAM LIMIT

Table 1 summarises the maximum measured beam-beam parameter in LEP for different energies and transverse damping times. The beam-beam limit was encountered at 45.6 GeV and was not reached for the highest beam energies. This allowed running with a four times smaller vertical emittance at 98 GeV, if compared to 45.6 GeV (taking into account beta functions and horizontal emittance).

The LEP data for 94.5 GeV to 101 GeV consistently suggest a beam-beam limit of around 0.115. Comparing this to the measured beam-beam limit of 0.045 at 45.6 GeV we find a scaling of the beam-beam limit as:

$$\xi_y^\infty \propto \lambda_d^{\sim 0.4}$$

This is close to the scaling suggested by Peggs [9].

5 CONCLUSION

A simple stochastic model of the beam-beam interaction was used to determine the expected functional dependence of the vertical beam-beam parameter on beam current. The derived equation depends on only two free parameters: the product of unperturbed emittances (at zero intensity) and the beam-beam limit. The expected functional dependence was fitted to many sets of LEP data. The measured saturation of the beam-beam parameter with current is well described. The fit was used to estimate the beam-beam limit for LEP at highest energies (around 0.11) and the best achievable luminosity. LEP is not reaching the beam-beam limit at high energy with vertical beam-beam parameters of up to 0.083 per IP. Vertical emittance blow-up is, however, observed both in the measured luminosity and the measured vertical beam size. The beam-beam limit seems to scale with $\lambda_d^{0.4}$ for LEP, with λ_d being the damping decrement.

REFERENCES

[1] S. Myers, "Simulation of the Beam-Beam Effect for e^+e^- Storage Rings". Nucl. Instr. Meth. 211 (1983) p. 263.

[2] S. Myers, "Review of Beam-Beam Simulations". In Santa Margherita Di Pula 1985, Proc. Nonlinear Dynamics Aspects Of Part. Acc., p. 176-237.

[3] E. Keil, "Beam-Beam Simulations for LEP at 87 GeV". CERN-SL-96-08-AP.

[4] H. Burkhardt et al, "The Effect of the Beam-Beam Interaction on the Performance of LEP". CERN-SL-92-15.

[5] H. Burkhardt, "Energy Dependence of Beam-Beam Interactions in LEP". Proc. PAC97.

[6] A. Chao, "Beam-Beam Instability", in Phys. of High Energy Part. Acc., AIP Conf. Proc. 127(1983).

[7] D. Brandt et al, "Is LEP Beam-Beam Limited?". Proc. PAC99.

[8] E. Keil and R. Talman, "Scaling of Luminosity Data between e+e- Storage Rings". Part. Acc. 14(1983) p.109.

[9] S. Peggs. Talk at the Workshop on Beam-Beam Effects (LHC99), CERN 1999.

[10] R. Assmann et al, "Luminosity and Beam Measurements Used for Performance Optimisation in the LEP Collider". These proceedings.

[11] A. Verdier. "Beam-Beam Effect in LEP, an Alternative Point of View". Beam Physics Note 28. CERN.

* Status 15.6.2000.