A Very Large Lepton Collider in the VLHC Tunnel

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http://wwwslap.cern.ch/∼keil/Doc/VLLC/09mar01.pdf

Programme

- VLLC parameter searches with a notebook and packages in *Mathematica*
  - `vllc.nb` is specific to VLLC
  - Packages contain formulae valid for many machines and e^+e^-, $\mu^+\mu^-$, p
  - `vllc.nb` finds parameters of VLLC arcs in thin-element approximation, synchrotron radiation, RF system, collective effects, etc.
  - `vllc.nb` writes short file with parameters for MAD
- MAD finds solutions for finite elements, corrects chromaticity, tracks, etc.
- Verify VLLC parameters of Sen and Norem
- Would I design a VLLC in the same manner?

Mathematica Input Data

`collE, injE`	Collision, injection energy in `Volt`
ρ, `R`	Bending, average radius of arc cells in `Meter`
`cirC`	Circumference in `Meter`
`phaseAdv`	Phase advance in arc cells/2π
`superN`	Number of superperiods
ξ	Beam-beam tune shift parameter
β`IPx`, β`IPy`	Amplitude functions at IP in `Meter`
`bunchK`	Number of bunches/beam
`totL`	Design luminosity in $\mathtt{Meter}^{-2}\mathtt{Second}^{-1}$
`fRF`	Approximate frequency of RF system in `1/Second`
`tauQ`	Quantum lifetime in `Hour`

- Note *Mathematica* dimensions
- A few parameters for wigglers and collective effects missing

Collider Parameter Table from *Mathematica*

*collE	184000000000	Volt
$*\rho$	25411	Meter
*R	30444	Meter
*cirC	228000	Meter
γ	360080.	
β	1.	
totArcL	191285	Meter
revolF	1314.88	1/Second
*bunchK	89	
bunchS	2561.8	Meter
dipoleB	0.0241532	Second Volt/Meter^2

Asterisks ($*$) mark input parameters

IP Parameter Table from *Mathematica*

$*\xi_x = *\xi_y$	0.1	
$*\beta$IPx, $*\beta$IPy	1., 0.05	Meter
bunchN	$6.36895 \cdot 10^{11}$	
bunchI	0.000134173	Ampere
ϵ_x	0.00272039	Meter
ϵ_y	0.000136019	Meter
σ_{IPx}	0.0000869192	Meter
σ_{IPy}	$4.34596 \cdot 10^{-6}$	Meter
σ'_{IPx}	0.0000869192	
σ'_{IPy}	0.0000869192	
lumic	$8.54524 \cdot 10^{31}$	Meter^{-2}
lumib	$1.1236 \cdot 10^{35}$	Meter^{-2}/Second
beamI	0.0119414	Ampere
checkL	$1. \cdot 10^{37}$	Meter^{-2}/Second

Arc Parameter Table from *Mathematica*

*phaseAdv	0.25	
periodN	856	
Q	214.	
periodL	223.464	Meter
periodA	0.00734017	
Av.β_x	142.262	Meter
Av.D_x	0.664774	Meter
maxβx	381.477	Meter
maxDx	1.11009	Meter
focalL	79.0065	Meter
η	0.0000216594	
quadL	0.650424	Meter

Wiggler Parameter Table from *Mathematica*

*epsF	1.01	
plusL	0.207465	Meter
sigeF	1.00116	
lossF	1.00023	
polF	1.00256	
tauP	3.03691	Hour
σ_e	0.000989889	
Av.σ_x	0.00122793	Meter
Max.σ_x	0.00202226	Meter
quadG	11.9437	Second Volt Meter^{-3}

Synchrotron Radiation Table from *Mathematica*

lossU	$3.9903827 \cdot 10^9$	Volt
powerSR	$4.7650645 \cdot 10^7$	Ampere Volt
powerSRm	249.10773	Ampere Volt/Meter
τ_x	0.070120636	Second
critE	543817.17	Joule/Coulomb
totPhotons	$9.2850791 \cdot 10^{15}$	1/(Meter Second)
distSR	49.350960	Meter
incidenceA	0.0016210406	
spotSR	0.00038486988	Meter
powSRm2	647251.82	Ampere Volt/Meter^2

RF System Table from *Mathematica*

hRF	304113	
lambdaRF	0.749721	Meter
fRF	3.99872	1/Second
bucketH	0.00599165	
overV	1.07053	
voltRF	$4.27281 \cdot 10^9$	Volt
ϕ_s	0.308093	
Q_s	0.0932196	
σ_s	0.00834603	Meter
bunchArea	0.0637196	Second Volt
*tauQ	24	Hour

MAD Data File

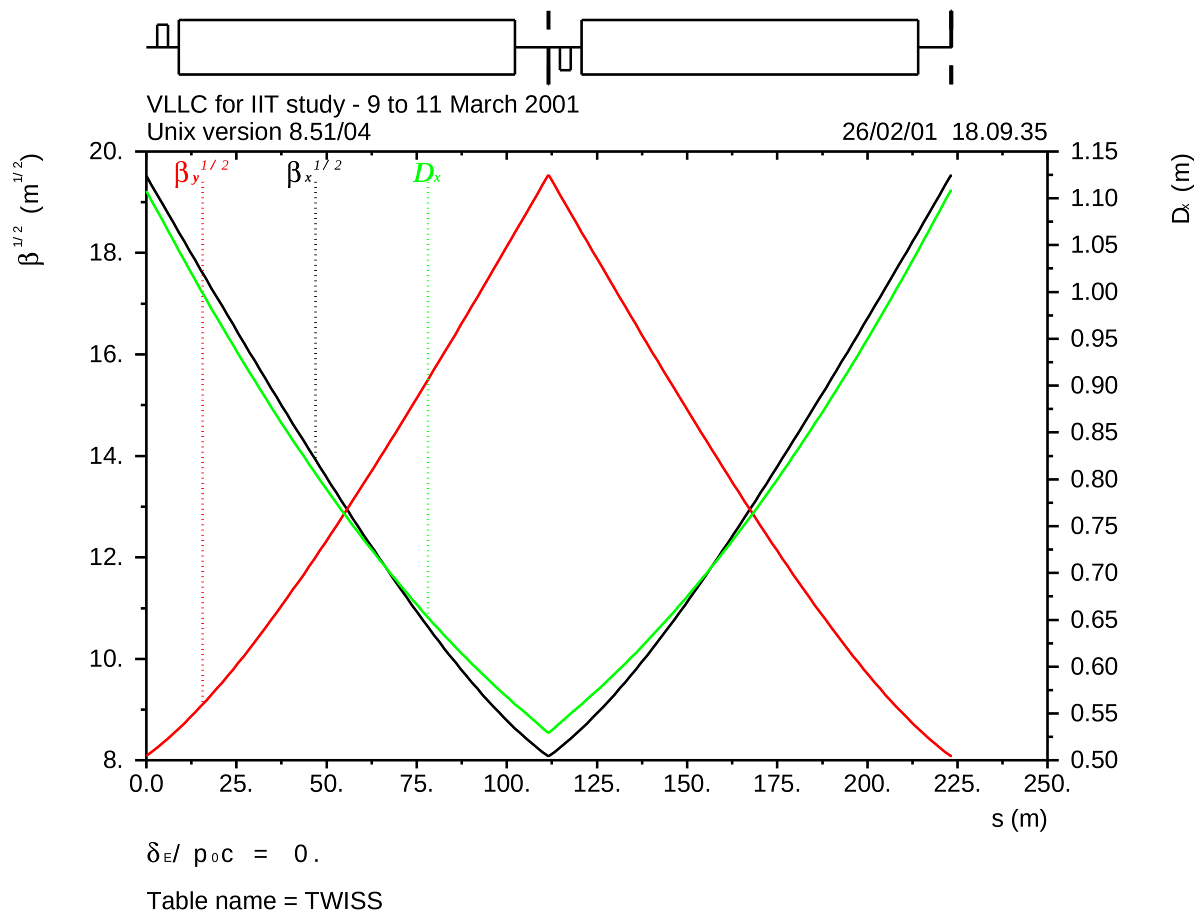
```
!-- Define arc parameters
mu2pi:=0.25
arcR:=30444
rho:=25411
cirC:=228000
periodN:=856
periodL:=223.46412791095247
dipoleL:=93.26052677613345
focalL:=79.00650009888628
quadL:=0.39494145390137947
freeS:=18.07659572544l408
return
```

Cell Definition in MAD

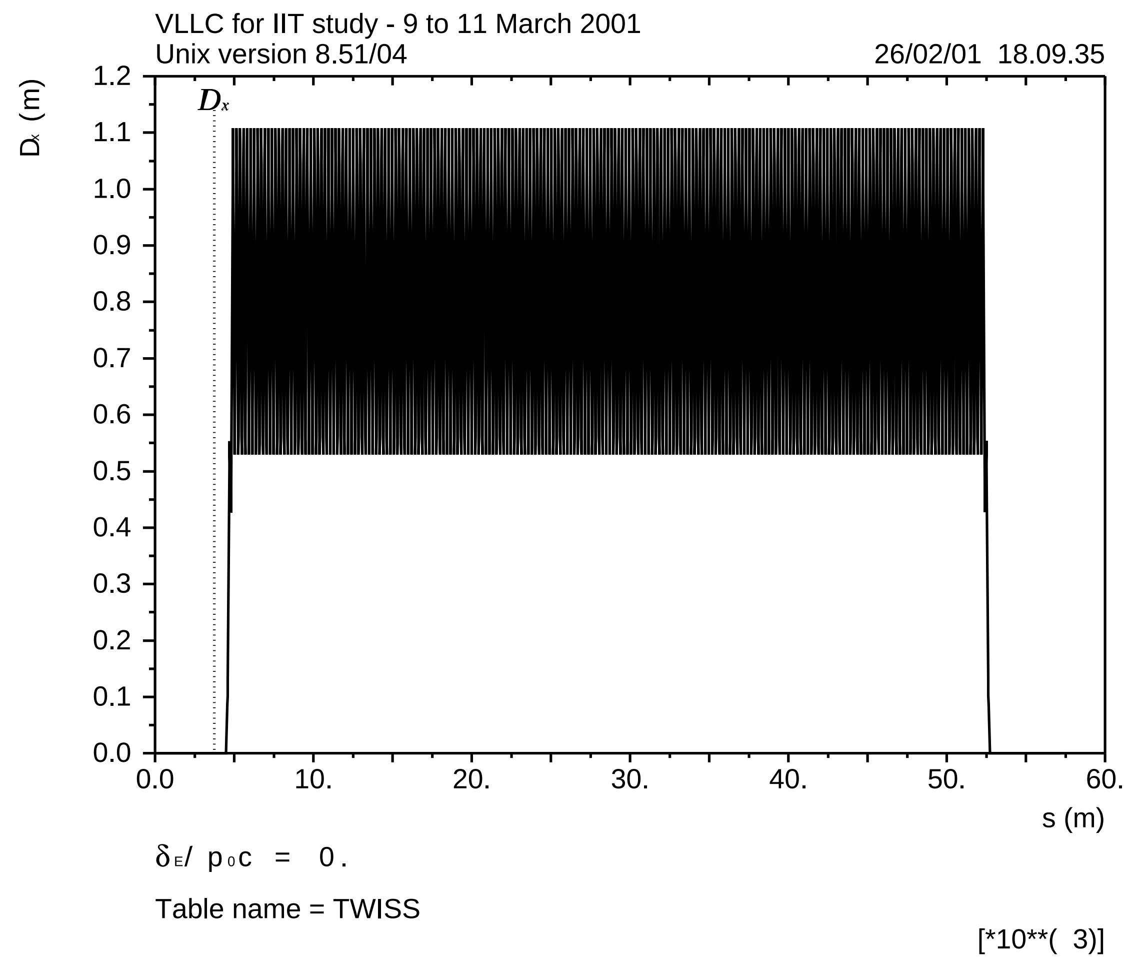
```
qfa: quad l=quadL k1=1/focalL/quadL
qda: quad l=quadL k1=-1/focalL/quadL
la: drift l=freeS/2; lsa: drift l=freeS/6
sextL:=freeS/6
sfa: sext l=sextL k2=+0.1; sda: sext l=sextL k2=-0.1
set v1 1/4/sin(pi*mu2pi)^2; set v2 (1-1/4/sin(pi*mu2pi)^2)
ba: rbend l=dipoleL, &
    angle=(2*pi)/(4*superN*(v1+v2)+2*(periodN-2*superN))
cold: rcollimator xsize=xcoldA ysize=ycoldA
colf: rcollimator xsize=xcolfA ysize=ycolfA
ca: line=(lsa,sfa,lsa,ba,la,cold,qda,cold, &
          lsa,sda,lsa,ba,la,colf,qfa,colf)
```

Note that cell definition is practically “number-free”

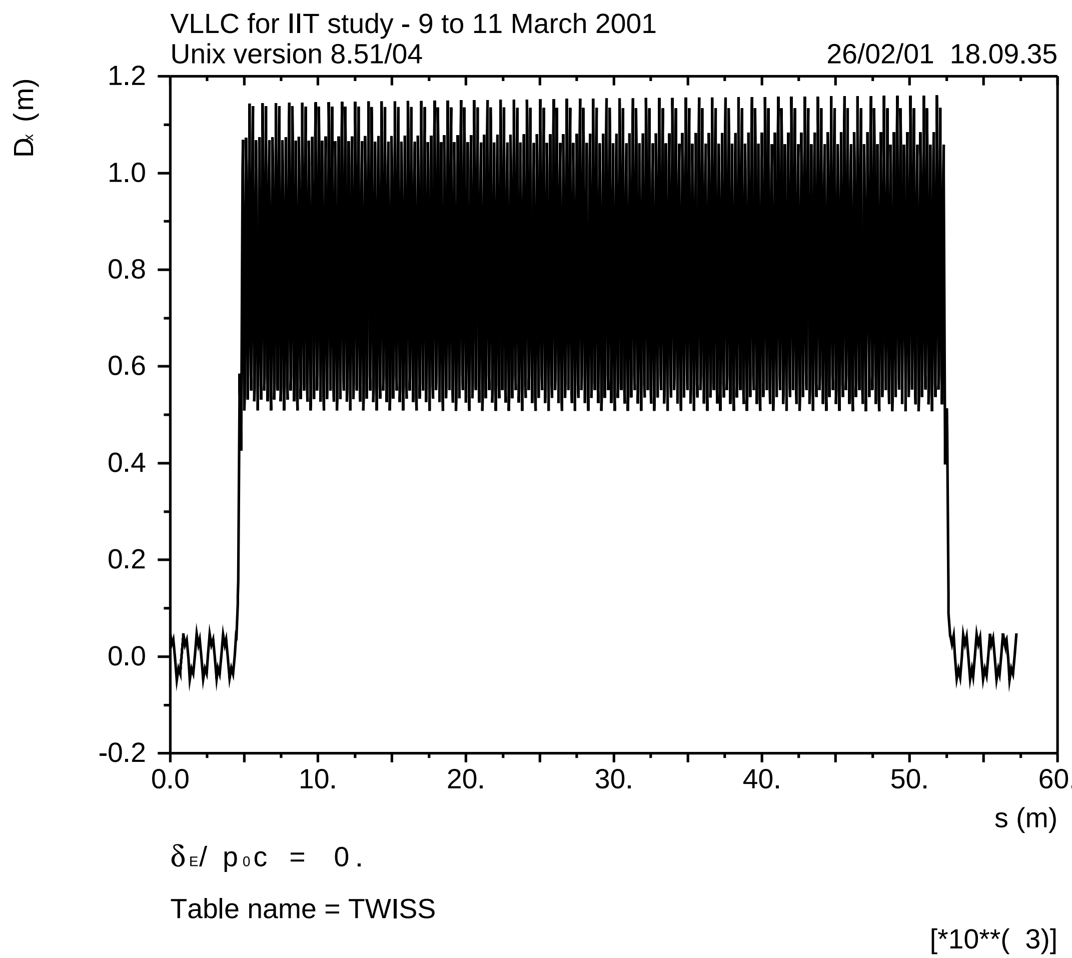
Arc Cell Parameters



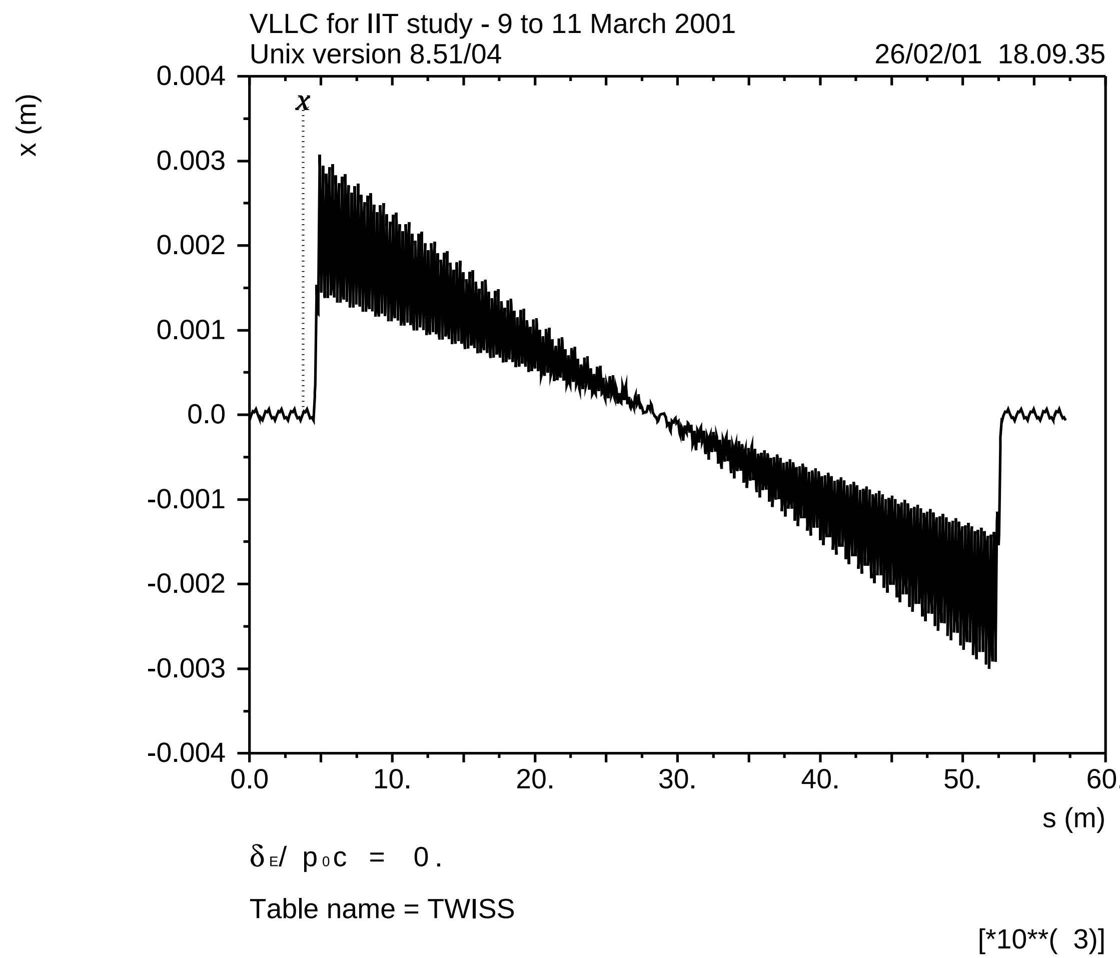
Horizontal Dispersion without Synchrotron Radiation



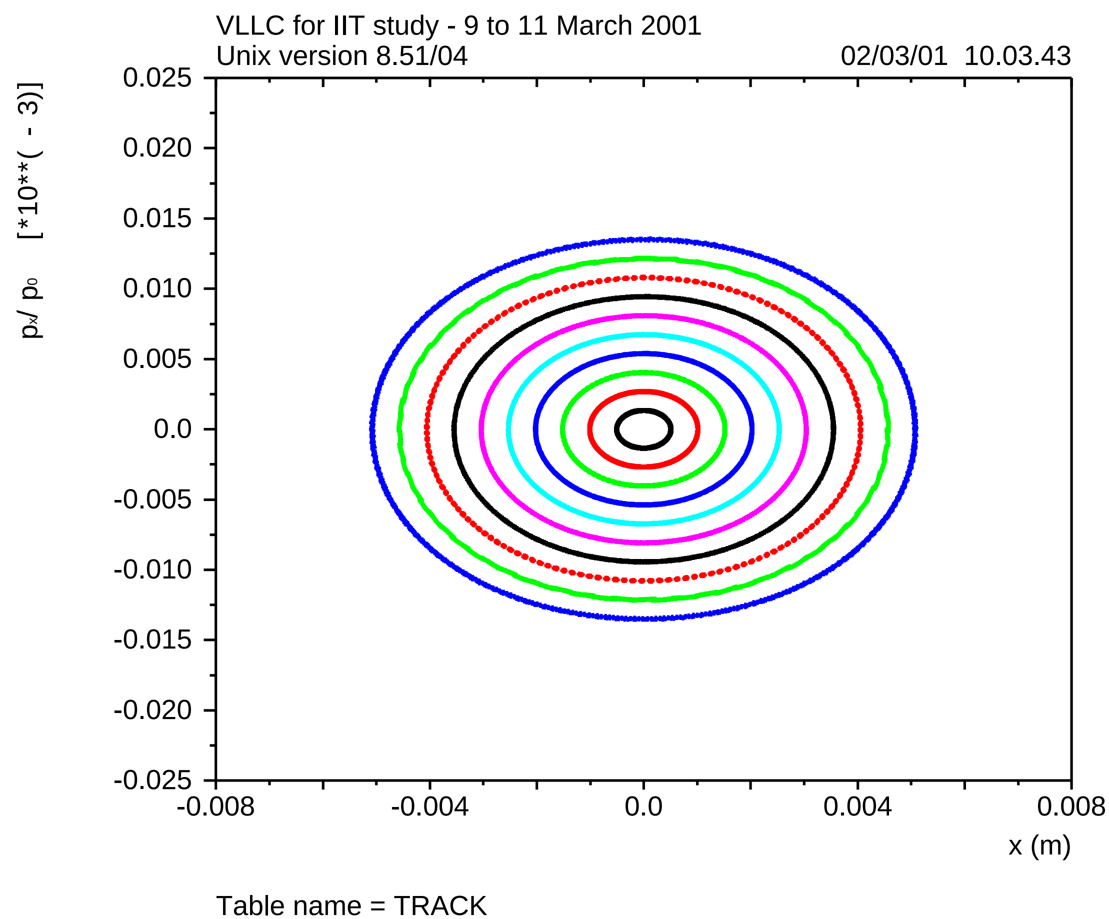
Horizontal Dispersion with Synchrotron Radiation



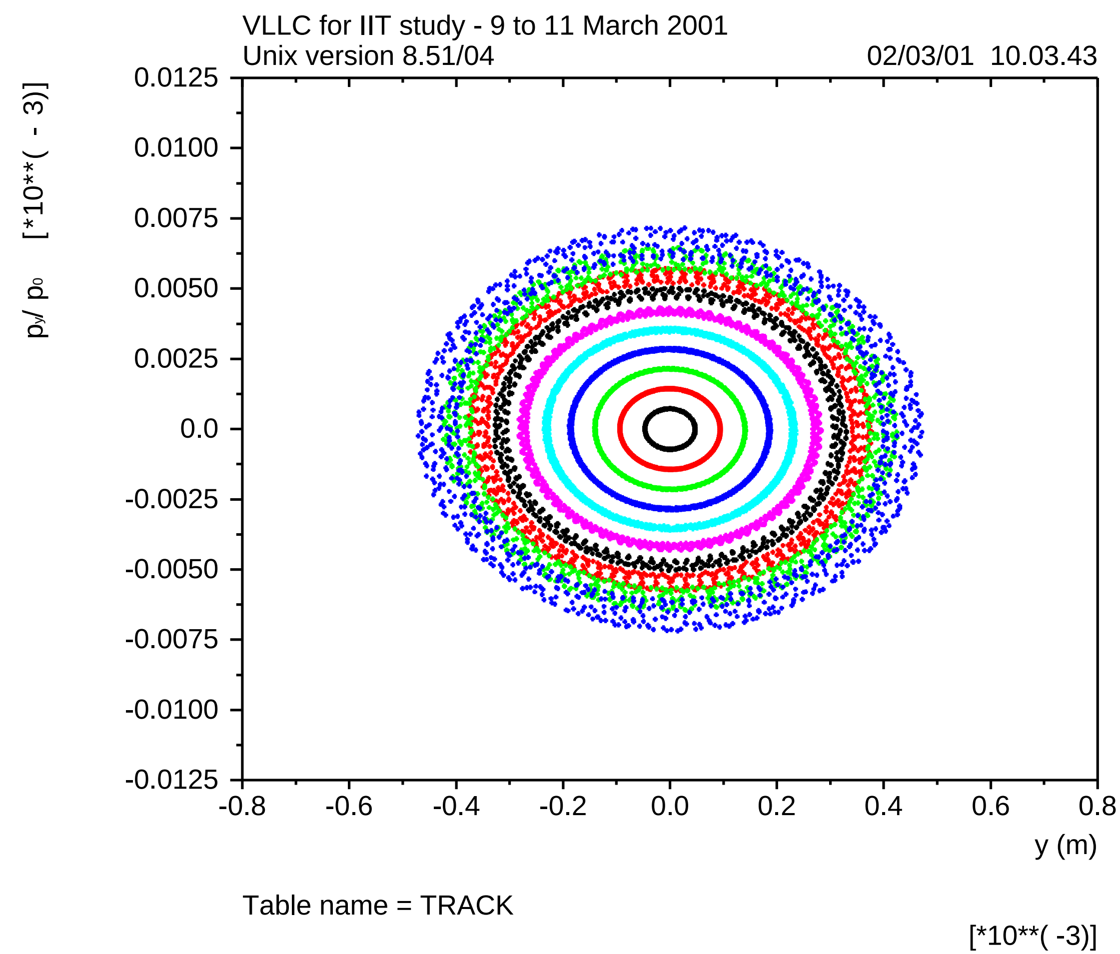
Horizontal Orbit Offset due to Synchrotron Radiation



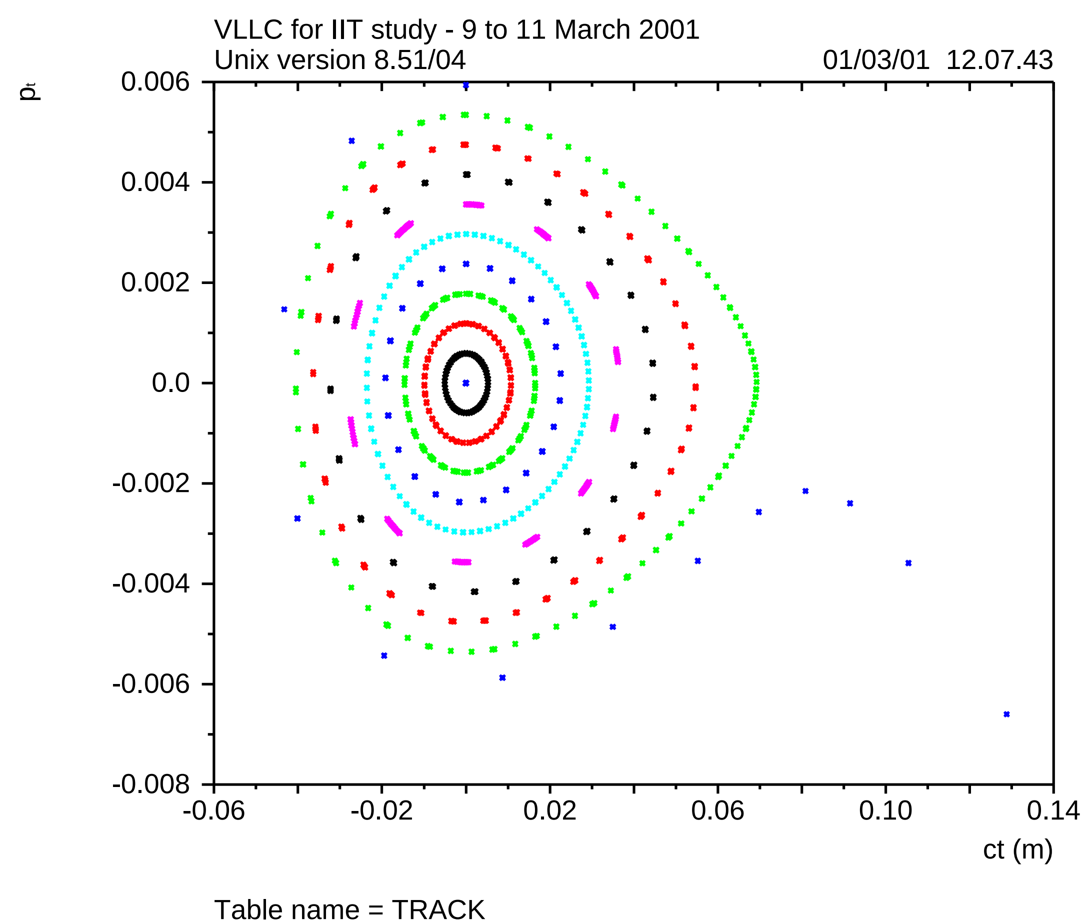
Horizontal Tracking Results: 10 e^+ up to $3\sigma_x$ – no SR – 10^3 turns



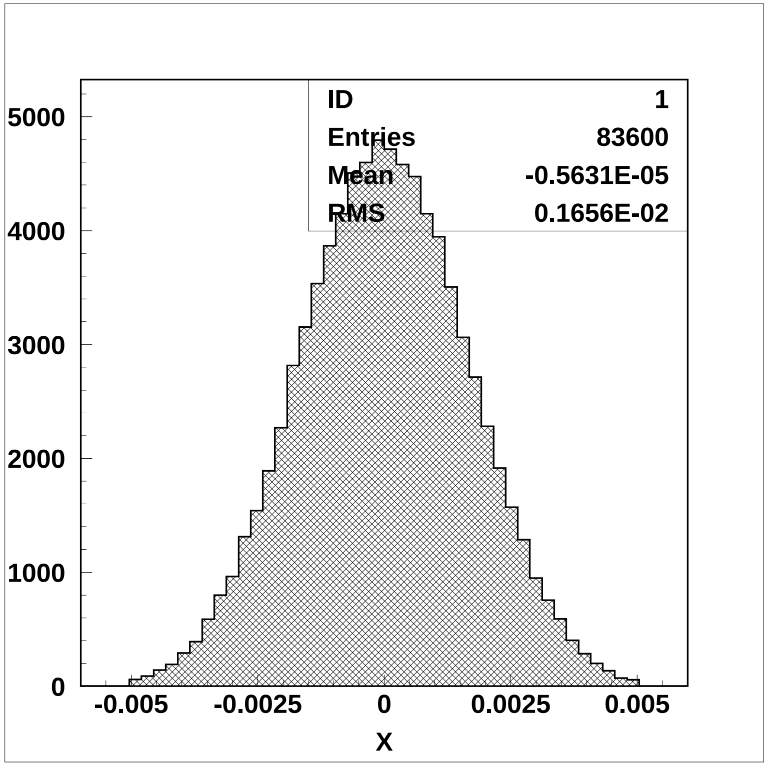
Vertical Tracking Results: 10 e$^+$ up to $3\sigma_y$ – no SR – 10^3 turns

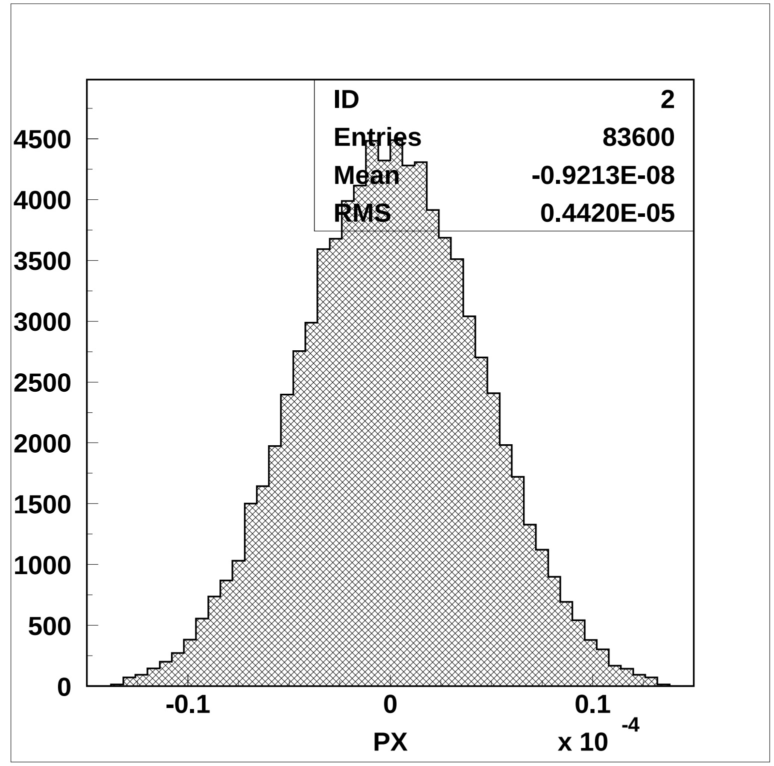


Long. Tracking Results: 10 e$^+$ up to $6\sigma_e$ – no SR – 100 turns



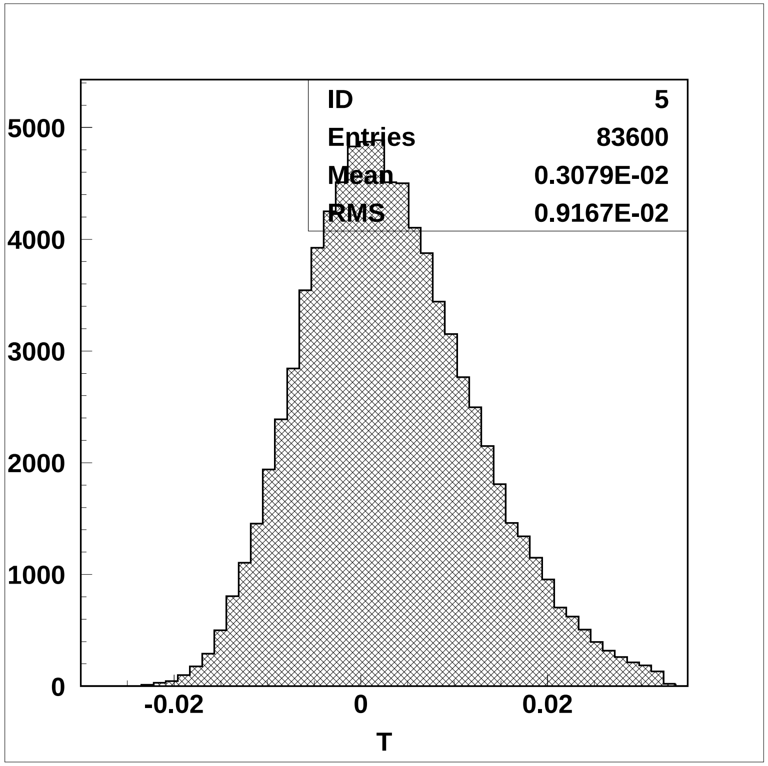
Horizontal Tracking Results: 300 e^+ – SR – 300 turns

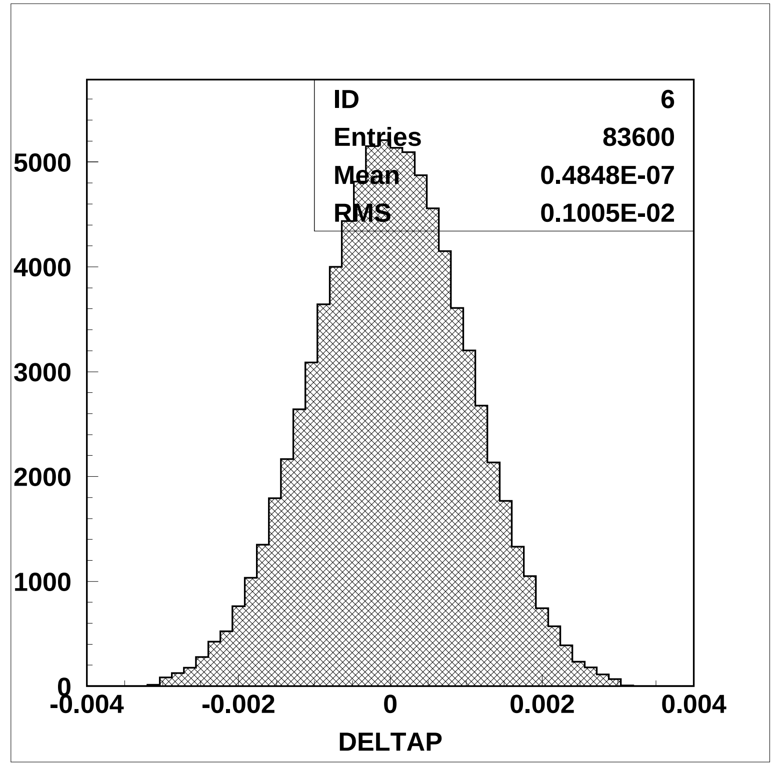




	σ_x/mm	σ'_x/μrad
Mathematica	1.69766	4.45022
MAD envelope	1.63622	4.351

Longitudinal Tracking Results: 300 e^+ – SR – 300 turns





	σ_t/mm	$\sigma_e/10^{-3}$
Mathematica	8.34438	0.989889
MAD envelope	8.565858	0.994076

Wiggler Magnets

- Homogeneous-field dipoles in $-++-$ arrangement
- In $+$ wigglers field `plusB` ≈ 0.3 T in same direction as in arc dipoles
- Compute total length `plusL` of $+$ poles
- Polarization requires field in $-$ wigglers a factor `ratioB` smaller
- Wigglers effective in VLLC with low B in arc dipoles
- **Always** increase energy spread σ_e and bunchlength σ_s, decrease damping times τ
- Wigglers may be “photon cutters”
- **Damping wigglers** installed at $D_x = 0$
  - reduce equilibrium emittance ϵ_{xn}, e.g. in collision below nominal current
  - Used at injection with `plusL` ≈ 340 m achieve $\sigma_{ei} \approx \sigma_e \approx 10^{-3}$
- **Emittance wigglers** installed at $D_x \neq 0$
  - increase equilibrium emittance ϵ_{xn}
  - best in dispersion bumps with $\mathcal{H}_{\mathrm{w}} \approx 4\mathcal{H}_{\mathrm{arc}}$, ratio is called `hWbyhB`
  - used in VLLC below maximum energy

VLLC Operation at 45 GeV with Wiggler Magnets

*lowepsF	100	
*lowhWbyhB	4	
lowplusL	66.5216	Meter
lowtauP	60.9516	Hour
σ_{ec}	0.00121707	
Max.σ_{xc}	0.00432691	Meter
bunchNc	$9.13202 \cdot 10^{11}$	
σ_{IPxc}	0.000210459	Meter
σ_{IPyc}	0.0000105229	Meter
beamIc	0.0171219	Ampere
totLc	$3.50665 \cdot 10^{36}$	Meter^{-2} Second^{-1}

Collective Effects: Transverse Mode Coupling Instability TMCI

- Bunch current threshold I_b with synchrotron frequency f_s, $e^{\pm}$ voltage E/e, transverse β and transverse loss factors $k_\perp(\sigma_s)$

$$I_b = \frac{8 f_s (E/e)}{\sum_i \beta_i k_\perp(\sigma_s)_i}$$

- Scale RF cavities by number from LEP with $k_\perp = 9.31$ V/pC at $\sigma_s = 10$ mm
- Find $I_b \approx 1.26$ mA at 184 GeV, and $I_b \approx 0.14$ mA at 20 GeV with my arrangement of damping wigglers at injection
- Scale “shielded bellows” from LEP with 10 m spacing and $k_\perp = 0.42$ V/pC at $\sigma_s = 10$ mm
- Find $I_b \approx 0.136$ mA at 184 GeV, and $I_b \approx 0.0148$ mA at 20 GeV with my arrangement of damping wigglers at injection
- TMCI dominated by bellows and below design current by factor 10